A User's Guide to Vacuum Technology

Third Edition

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Third Edition

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For Jean, Carol, Paul, and Amanda

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Preface

This book is intended for the vacuum system user—the university student, technician, engineer, manager, or scientist—who wishes a fundamental understanding of modern vacuum technology and a user's perspective of modern laboratory and industrial vacuum technology.

Vacuum technology is largely secondary; it forms part of other technologies that are central to analysis, research, development, and manufacturing. It is used to provide a process environment. Many advances in vacuum technique have resulted from the demands of other technologies, although scientists and engineers have studied vacuum for its own sake. The average user is process-oriented and becomes immersed in vacuum technique only when problems develop with a process or when new equipment purchases become necessary.

A *User's Guide to Vacuum Technology, 3rd Edition* focuses on the operation, understanding, and selection of equipment for processes used in semiconductor, optics, and related technologies. It emphasizes subjects not adequately covered elsewhere, while avoiding in-depth treatments of topics interesting only to the designer or curator. Residual gas analysis is an important topic whose treatment differs from the usual explanation of mass filter theory. Components such as the turbomolecular and helium gas refrigerator cryogenic pumps are now widely used but not well understood. The discussion of gauges, pumps, and materials is a prelude to the central discussion of systems. System designs are grouped according to their function. Current designs are either single-chamber or multichamber; the details of each design are determined by the requirements of an industrial or research application.

In this edition, the discussion of gauges, pumps, and materials has been updated, where relevant, to reflect changes in practice. Spinning rotor gauges are no longer a laboratory curiosity. Ultrahigh vacuum gauges, though limited in their availability, will be a necessity in next-generation production deposition systems. Ultraclean, low dead volume metrology and valves, along with superior materials and cleaning techniques, have made contamination-free manufacturing a reality.

Ultraclean vacuum, once the domain of the researcher, is now routinely used for high-volume production of semiconductor chips and storage

media. However, methodologies for reaching low pressures in a clean manner have changed significantly. No longer are single-chamber systems baked for twenty-four hours. Rather, cassette-based load/unload chambers serve as high-volume interfaces between atmosphere and ultraclean process chambers. These chambers, which can be accessed in serial or random order, are only exposed to atmosphere during maintenance.

Large, efficient multichamber medium and highvacuum systems are used in high-speed coating of numerous consumer products such as window glass, solar cells, video tape, printer paper, eyeglass lenses, automobile headlamps, plastic films and security devices.

The gap in knowledge and training between those who manufacture and those who use vacuum equipment continues to widen. It is from this perspective that the previous edition of this book has been revised. Important formulas have been denoted with a ▶ for emphasis. Easy questions have been emphasized with a †.

Thanks are due to countless researchers who, individually and collaboratively, have advanced this field by creative solutions to real problems; I also thank Dr. Bruce Kendall for his insightful comments and thoughtful review.

J. F. O'Hanlon

Tucson, Arizona

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Its Basis

An understanding of how vacuum components and systems function begins with an understanding of the behavior of gases at low pressures. Chapter 1 discusses the nature of vacuum technology. Chapter 2 reviews basic gas properties. Chapter 3 describes the flow of gases at reduced pressures, and Chapter 4 discusses how gas is evolved from the surfaces of materials. Together, these chapters form the basis of vacuum technology.

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CHAPTER 1

Vacuum Technology

Torricelli is credited with the conceptual understanding of the vacuum within a mercury column by 1643. It is written that his good friend Viviani actually performed the first experiment, perhaps as early as 1644 [1,2]. His discovery was followed in 1650 by Otto von Guericke's piston vacuum pump. Interest in vacuum remained at a low level for more than 200 years, when a period of rapid discovery began with McLeod's invention of the compression gauge. In 1905 Gaede, a prolific inventor, designed a rotary pump sealed with mercury. The thermal conductivity gauge, diffusion pump, ion gauge, and ion pump soon followed, along with processes for liquefaction of helium and refinement of organic pumping fluids. They formed the basis of a technology that has made possible everything from light bulbs to space simulation. The significant discoveries of this early period of vacuum science and technology have been summarized in a series of historical review papers [2–7].

A vacuum is a space from which air or other gas has been removed. All gas cannot be removed. The amount removed depends on the application, and is done for many reasons. At atmospheric pressure molecules constantly bombard surfaces. These molecules can bounce from surfaces, attach themselves to surfaces, or perhaps chemically react with surfaces. Air or other surrounding gas quickly contaminates a cleaned surface. A clean surface—for example, a freshly cleaved crystal—will remain clean in an ultrahigh vacuum chamber for long periods of time, because the rate of molecular bombardment is low.

Molecules are crowded closely together at atmospheric pressure and travel in every direction much like people in a crowded plaza. It is impossible for a molecule to travel from one wall of a chamber to another without colliding with many molecules. By reducing the pressure to a suitably low value, a molecule from one wall can travel to another without a collision. Many effects become possible if molecules can travel long distances between collisions. Metals can be evaporated from a pure source without reacting in transit. Molecules or atoms can be accelerated to a high

energy and sputter away, or be implanted in the bombarded surface. Electrons or ions can be scattered from surfaces and be collected. The energy changes they undergo on scattering or release from a surface can be used to probe or analyze the surface or underlying layers. For convenience the subatmospheric pressure scale has been divided into several ranges. Table 1.1 lists these ranges.

The required vacuum level depends on the application. Epitaxial growth of semiconductor films (reduced pressure epitaxy) and laser etching of metals are two processes that are performed in the low vacuum range. Sputtering, plasma etching and deposition, low-pressure chemical vapor deposition, ion plating, and gas filling of encapsulated heat transfer modules are examples of processes performed in the medium vacuum range.

Pressures in the high vacuum range are needed for the manufacture of traditional low- and high-tech devices such as microwave, power, cathode ray and photomultiplier tubes, light bulbs, architectural and automotive glazing, decorative packaging, degassing of metals, vapor deposition, and ion implantation. A number of medium technology applications including medical, microwave susceptors, electrostatic dissipation films, and aseptic packaging use films fabricated in a vacuum environment [8]. Retail security, bank note security, and laser and inkjet paper have joined this group.

The background pressure must be reduced to the very high vacuum range for electron microscopy, mass spectroscopy, crystal growth, and x-ray and electron beam lithography, and storage media production. For ease of reading, we call the very high vacuum region "high vacuum" and call the pumps "high vacuum pumps."

Pressures in the ultrahigh vacuum range were formerly the domain of the surface analyst, materials researcher, or accelerator technologist. Critical high-volume production applications, such as semiconductor devices, thin-

Table 1.1 Vacuum Ranges

Degree of Vacuum	Pressure Range (Pa)[a]
Low	$10^{5} > P > 3.3\times10^{3}$
Medium	$3.3\times10^{3} \geq P > 10^{-1}$
High	$10^{-1} \geq P > 10^{-4}$
Very high	$10^{-4} \geq P > 10^{-7}$
Ultrahigh	$10^{-7} \geq P > 10^{-10}$
Extreme ultrahigh	$10^{-10} > P$

Source: Reprinted with permission from *Dictionary for Vacuum Science and Technology*, M. Kaminsky and J. M. Lafferty, Eds., American Vacuum Society, New York, 1980.
[a] 101323.3 Pa = 1 atmosphere.

film media heads, and extreme UV lithography systems, require ultrahigh vacuum base pressures to improve yield by reducing gaseous impurity contamination. Additionally, processes carried out in these systems must be free of particle contamination, so we call them ultraclean vacuum systems.

A vacuum system is a combination of pumps, valves, and pipes, which creates a region of low pressure. It can be anything from a simple mechanical pump or aspirator for exhausting a vacuum storage container to a complex system such as an underground accelerator with miles of piping that is maintained at ultrahigh vacuum.

Removal of air at atmospheric pressure is usually done with a displacement pump. A displacement pump is one that removes the air from the chamber and expels it to the atmosphere. Rotary vane and piston pumps are examples of pumps used to exhaust gases at atmospheric pressure. Liquid nitrogen capture pumps or sorption pumps have also been designed for exhausting gases at atmospheric pressure. They are used only on small chambers because of their finite gas sorption.

Rotary vane, piston and sorption pumps have low-pressure limits in the range 10^{-1}–10^{-3} Pa. Pumps that will function in a rarefied atmosphere are required to operate below this pressure range. Several displacement and capture pumps can remove air at these low pressures. The diffusion pump was the first high vacuum pump. It is a displacement pump. Its outlet pressure is below atmosphere. The turbomolecular pump, a system of high-speed rotating turbine blades, can also pump gas at low pressures. The outlet pressures of these two pumps need to be kept in the range 0.5–50 Pa, so they must exhaust into a rotary vane or piston "backing" pump, or "fore" pump. If the diffusion or turbomolecular pump exhaust gas flow would otherwise be too great, a lobe blower will be placed between the exhaust of the diffusion or turbomolecular pump and the inlet of the rotary pump to pump gas at an increased speed in this intermediate pressure region.

Capture pumps can effectively remove gas from a chamber at low pressure. They do so by freezing molecules on a wall (cryogenic pump), chemically reacting with the molecules (getter pump), or accelerating the molecules to a high velocity and burying them in a metal wall (ion pump). Capture pumps are more useful as high vacuum pumps than as atmospheric exhaust pumps because the number of molecules to be captured at high vacuum is less than the number removed during initial evacuation from atmosphere.

Air is the most important gas to understand, because it is in every vacuum system. It contains at least a dozen constituents, whose major constituents are described in Table 1.2. The differing ways in which pumps remove air, and gauges measure its pressure, can be understood in terms of the partial pressures of its components. The concentrations listed in Table 1.2 are those of dry atmospheric air at sea level (total pressure

Table 1.2 Components of Dry Atmospheric Air

Constituent	Content (vol. %)	Content (ppm)	Pressure (Pa)
N_2	78.084 ± 0.004		79,117
O_2	20.946 ± 0.002		21,223
CO_2 [a]	0.037		37.5
Ar	0.934 ± 0.001		946.357
Ne		18.18 ± 0.04	1.842
He		5.24 ± 0.004	0.51
Kr		1.14 ± 0.01	0.116
Xe		0.087 ± 0.001	0.009
H_2		0.5	0.051
CH_4		2.	0.203
N_2O		0.5 ± 0.1	0.051

Source: Reprinted with permission from *The Handbook of Chemistry and Physics*, 59th ed., R. C. Weast, Ed., copyright 1978, The Chemical Rubber Publishing Co., CRC Press, Inc., West Palm Beach, FL 33409.

[a] Carbon dioxide data from Mauna Kea, Hawaii, 2000. Data since 1955 are available as: http://stratus.mlo.hawaii.gov/Projects/GASES/co2graph.htm.

101,323.2 Pa or 760 Torr). The partial pressure of water vapor is not given in this table, because it constantly changes. At 20°C a relative humidity of 50% corresponds to a partial pressure of 1165 Pa (8.75 Torr), making it the third largest constituent of air. The total pressure changes rapidly with altitude, as shown in Fig. 1.1, whereas its proportions change slowly but significantly. In outer space the atmosphere is mainly H_2 with some He [6].

In the pressure region below 10 Pa, gases evolving from material surfaces contribute more molecules per second to the total gas load than do the gases originally filling the chamber. The correct pump is not the only requirement needed to reach low pressures—the materials of construction, techniques for joining components, surface cleaning techniques, and operational procedures are all critically important. In the remaining chapters the pumps, gauges, and materials of construction and operational techniques are described in terms of fundamental gas behavior. The focus is on the understanding and operation of vacuum systems for a variety of technological applications.

1.1 UNITS OF MEASUREMENT

Units of measurement present problems in many disciplines and vacuum technology is no exception. The use of noncoherent vacuum units has been common in the US long after the adoption of System International.

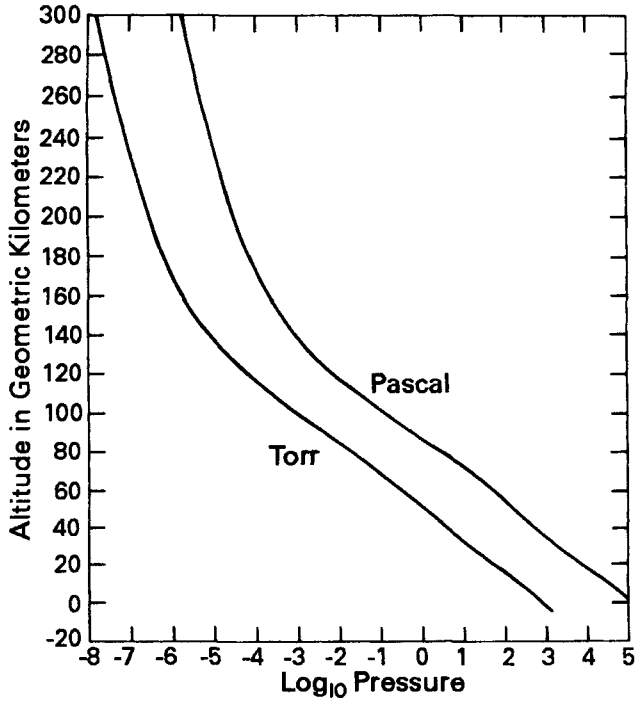

Fig. 1.1 Relation between the atmospheric pressure and the geometric altitude. Reprinted with permission from *The Handbook of Chemistry and Physics*, 59th ed., R. C. Weast, Ed. copyright 1978, The Chemical Rubber Publishing Co., CRC Press, Inc., West Palm Beach, FL 33409.

The meter-kilogram-second (MKS) system was first introduced over a half-century ago; its use became commonplace only after a decade or more of classroom education by instructors committed to change. In a similar manner, those who teach vacuum technique will lead the way to routine use of SI units. Instruments are manufactured for use in a global economy and their readings can be displayed in several formats. The advantages of using a coherent unit system are manifold. Calculations become straightforward and logical and the chance for error is reduced. Incoherent units such as permeation constant, the volume of gas (at standard temperature and pressure) per material thickness per material area per sec pressure difference, are cumbersome. Additionally, these permeation units mask their relation to solubility and diffusion. Ultimately, SI units will be routinely used. To assist with this change, dual labels have been added throughout the text. Basic SI units for pressure (Pa), time (s) and length (m) will be assumed in all formulas, unless noted differently within a formula statement.

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CHAPTER 2

Gas Properties

In this chapter we discuss the properties of gases at atmospheric and reduced pressures. The properties developed here are based on the kinetic picture of a gas. Kinetic theory has its limitations, but with it we are able to describe particle motion, pressure, effusion, viscosity, diffusion, thermal conductivity, and thermal transpiration of ideal gases. We will use these ideas as the starting point for discussing gas flow, gauges, pumps and systems.

2.1 KINETIC PICTURE OF A GAS

The kinetic picture of a gas is based on several assumptions. (*i*) The volume of gas under consideration contains a large number of molecules. A cubic meter of gas at a pressure of 10^5 Pa and a temperature of 22°C contains 2.48×10^{25} molecules, whereas at a pressure of 10^{-7} Pa, a very high vacuum, it contains 2.5×10^{13} molecules. Indeed, any volume and pressure normally used in the laboratory will contain a large number of molecules. (*ii*) Adjacent molecules are separated by distances that are large compared with their individual diameters. If we could stop all molecules instantaneously and place them on the coordinates of a grid, the average spacing between them would be about 3.4×10^{-9} m at atmospheric pressure (10^5 Pa). The diameter of most molecules is of order $2–6\times10^{-10}$ m and their separation distances are ~6–15 times their diameter at atmospheric pressures. For extremely low pressures, say 10^{-7} Pa, the separation distance is about 3×10^{-5} m. (*iii*) Molecules are in a constant state of motion. All directions of motion are equally likely and all velocities are possible, although not equally probable. (*iv*) Molecules exert no force on one another except when they collide. If this is true, then molecules will be uniformly distributed throughout the volume and travel in straight lines until they collide with a wall or with one another.

Using these assumptions, many interesting properties of ideal gases have been derived. Some elementary properties are reviewed here.

2.1.1 Velocity Distribution

As the individual molecules move about they collide with elastic collisions. Elastic collisions conserve energy, whereas the colliding particle's velocity is changed after each collision. We stated that all velocities are possible, but not with equal probability. The distribution of particle velocities calculated by Maxwell and Boltzmann is

$$\frac{dn}{dv} = \frac{2N}{\pi^{1/2}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/(2kT)} \qquad (2.1)$$

m is the particle mass and T is the Kelvin temperature. The relation between the Kelvin scale and the Celsius scale is $T(\text{K}) = 273.16 + T(°\text{C})$. In (2.1) N is the total number of particles, and k is Boltzmann's constant. Figure 2.1 illustrates (2.1) for nitrogen molecules (air) at three temperatures. It is a plot of the relative number of molecules between velocity v and $v + dv$. We see that there are no molecules with zero or infinite velocity, and that the peak or most probable velocity v_p, is a function of the average gas temperature. The particle velocity also depends on the molecular mass, the peak velocity can be expressed as $v_p = (2kT/m)^{1/2}$. The arithmetic mean or average velocity v is useful when describing particle flow.

$$v = \left(\frac{8kT}{\pi m}\right)^{1/2} \qquad \blacktriangleright (2.2)$$

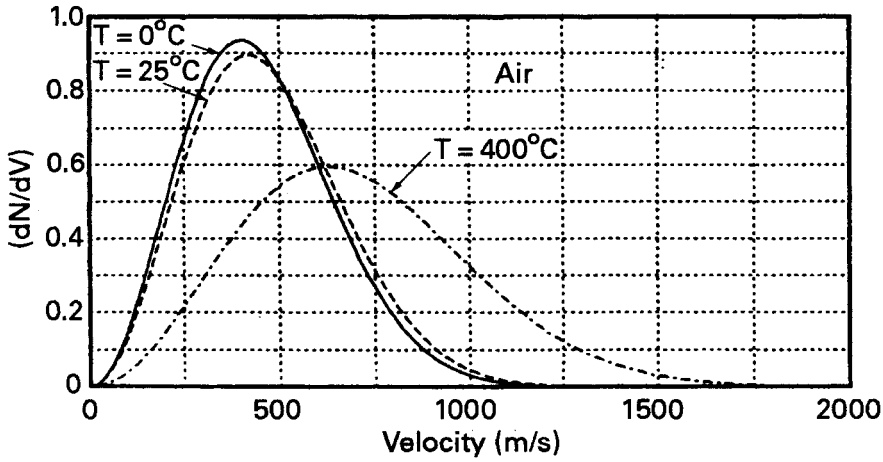

Fig. 2.1 Relative velocity distribution of air at 0°C, 25°C, and 400°C.

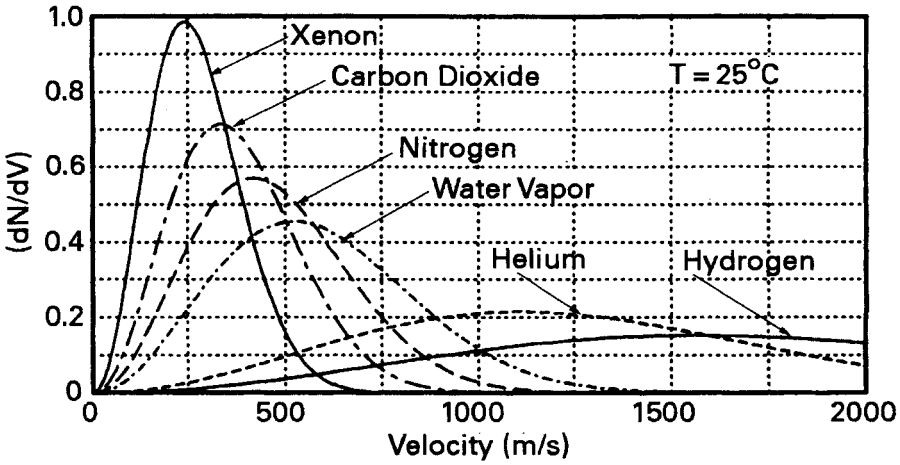

Fig. 2.2 Relative velocity distribution of several gases at 25°C.

The average velocities of several gas and vapor molecules are given in Appendix B.2. The root of the mean square velocity, is $v_{rms} = (3kT/m)^{1/2}$. The rms velocity is the square root of the average or mean of each velocity squared times the number of particles with that velocity. For Maxwell-Boltzmann statistics the average velocity is always 1.128 times as large as v_p, while $v_{rms} = 1.225v_p$. In Fig. 2.1 we illustrated the temperature dependence of the velocity distribution. As the temperature is increased the peak is broadened and shifted to a higher velocity. We may also plot (2.1) for different gases having the same temperature. Figure 2.2 illustrates the velocity distribution for H_2, He, H_2O, N_2, CO_2, and Xe. There are two concepts illustrated in Figs. 2.1 and 2.2. First the average velocity of a particle is proportional to $(T/m)^{1/2}$. An increase in temperature or decrease in mass causes an increase in a particle's velocity and the frequency with which it collides with other particles or nearby walls. Second, not all the particles in a distribution have the same velocity. The Maxwell-Boltzmann distribution is quite broad—over 5% of the molecules travel at velocities greater than two times the average velocity.

2.1.2 Energy Distribution

Maxwell and Boltzmann also derived an energy distribution, which is based on the same assumptions as the velocity distribution. It is

$$\frac{dn}{dE} = \frac{2N}{\pi^{1/2}} \frac{E^{1/2}}{(kT)^{3/2}} e^{-E/(kT)} \tag{2.3}$$

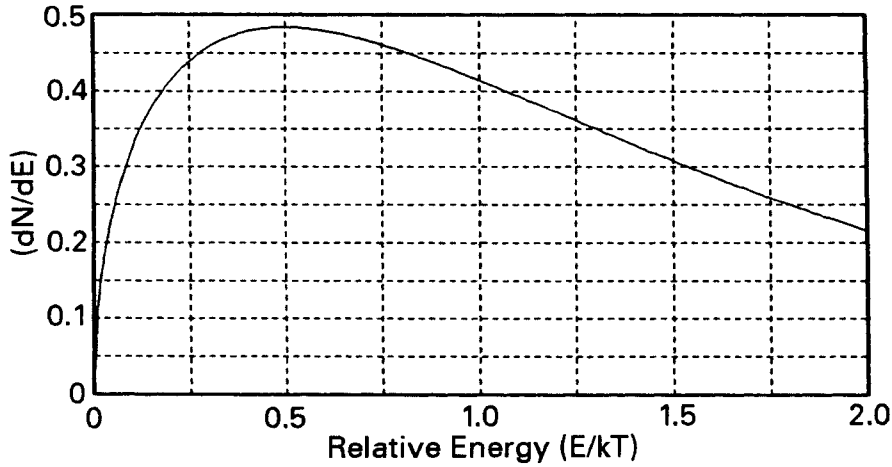

Fig. 2.3 Relative energy distribution of a gas at 25°C.

From this expression the average energy can be calculated as $E_{ave} = 3kT/2$, and the most probable energy as $E_p = kT/2$. Notice that neither the energy distribution nor the average energy is a function of the molecular mass. Each is only a function of temperature, as shown in Fig. 2.3. For example, all the gases depicted in Fig. 2.2 have the same energy distribution, because they are all have the same average temperature. See Fig. 2.3.

2.1.3 Mean Free Path

The fact molecules are randomly distributed and move with different velocities implies that each travels a different straight-line distance, known as a *free path* before suffering a collision. As illustrated in Fig. 2.4, not all free paths are the same length. The average, or mean, of the free paths λ, is found from kinetic theory

$$\lambda = \frac{1}{2^{1/2}\pi d_o^2 n} \qquad \blacktriangleright(2.4)$$

where d_o is the molecular diameter in meters, and n is the gas density in molecules per cubic meter. The mean free path is clearly gas density dependent. If the temperature is constant, it is also pressure dependent. See (2.12). For air at room temperature the mean free path is most easily remembered by one of the following expressions

$$\lambda\,(\text{cm}) = \frac{0.67}{P\,(\text{Pa})} \quad \text{or} \quad \lambda\,(\text{cm}) = \frac{0.005}{P\,(\text{Torr})} \qquad \blacktriangleright(2.5)$$

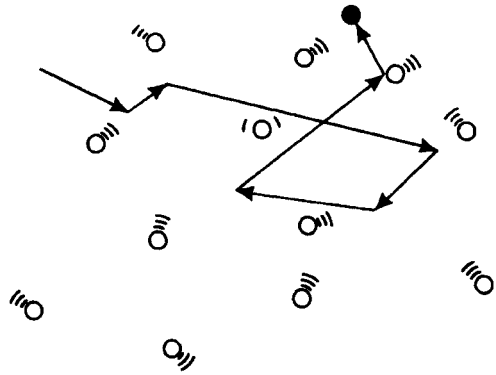

Fig. 2.4 Individual molecular paths.

where λ has units of cm, and P is the pressure in Pascal or Torr, respectively. Kinetic theory also describes the distributions of free paths.

$$N = N' e^{-x/\lambda} \tag{2.6}$$

N' is the number of molecules in the volume and N is the number of molecules that traverse a distance x before suffering a collision. Equation 2.6 states that 63% of the collisions occur in a distance $0 \leq x \leq \lambda$, whereas about 37% of the collisions occur in range $\lambda \leq x \leq 5\lambda$. Only about 0.6% of the particles travel distances greater than 5λ without suffering a collision.

For the case of two gases, a and b, the mean free path of a in b is

$$\lambda_a = \frac{1}{\left[2^{1/2}\pi n_a d_a^2 \left(1+\frac{v_b^2}{v_a^2}\right)^{1/2} n_b \frac{\pi}{4}(d_a+d_b)^2\right]} \tag{2.7}$$

2.1.4 Particle Flux

The concept of particle flux is helpful in understanding gas flow, pumping, and evaporation. According to kinetic theory the flux Γ of an ideal gas striking a unit surface or crossing an imaginary plane of unit area from one side is

$$\Gamma(\text{particles-m}^2\text{-s}^{-1}) = nv/4 \tag{2.8}$$

where n is the particle density and v, the average velocity. On substituting (2.2) we see that

$$\Gamma = n\left(\frac{kT}{2\pi m}\right)^{1/2} \tag{2.9}$$

The particle flux is directly proportional to the particle density and the square root of T/m.

2.1.5 Monolayer Formation Time

The time to saturate a surface with one layer of molecules is a function of the molecular arrival rate Γ, an molecular size. Assuming each molecule sticks and occupies surface area d_o^2, the time to form a monolayer is

$$t_{ml} = \frac{1}{\Gamma d_o^2} = \frac{4}{nvd_o^2} \tag{2.10}$$

At ambient temperature, a monolayer of air (d_o = 0.372 nm, v = 467 m/s) will form in about 2.5 s at a pressure of 10^{-4} Pa. The formation time will be longer, if the sticking coefficient is less than unity.

2.1.6 Pressure

The absolute pressure on a surface is defined as the rate at which momentum mv, is imparted to a unit surface. A molecule incident on a surface at an angle φ from the normal will impart a total impulse or pressure of $2mv \cos \varphi$. By integrating over all possible angles in the half-plane we find that the pressure is

$$P = \frac{1}{3} n m v_{\text{rms}}^2 \tag{2.11}$$

The total energy of a molecule, however, is proportional to its temperature

$$E = \frac{m v_{\text{rms}}^2}{2} = \frac{3}{2} kT \tag{2.12}$$

Equations (2.11) and (2.12) may be combined to form the ideal gas law.

$$P = nkT \tag{2.13}$$

If n is expressed in units of m^{-3}, k in joules per kelvin, and T in kelvin, then P will have units of pascal (Pa). A pascal is a newton per square meter and the fundamental unit of pressure in System International (SI). Simply divide the number of pascals by 133.32 to convert to units of Torr, or divide by 100 to convert to units of millibars. A conversion table is included in Appendix A.3. Values of n, d', λ, and Γ for air at 22°C are

Table 2.1 Low Pressure Properties of Air[a]

Pressure (Pa)	n (m^{-3})	d' (m)	λ (m)	Γ (m^{-2}-s^{-1})
1.01×10^{5} (760 Torr)	2.48×10^{25}	3.43×10^{-9}	6.5×10^{-8}	2.86×10^{27}
100 (.75 Torr)	2.45×10^{22}	3.44×10^{-8}	6.6×10^{-5}	2.83×10^{24}
1 (7.5 mTorr)	2.45×10^{20}	1.6×10^{-7}	6.6×10^{-3}	2.83×10^{22}
10^{-3} (7.5×10^{-6} Torr)	2.45×10^{17}	1.6×10^{-6}	6.64	2.83×10^{19}
10^{-5} (7.5×10^{-8} Torr)	2.45×10^{15}	7.41×10^{-6}	664	2.83×10^{17}
10^{-7} (7.5×10^{-10} Torr)	2.45×10^{13}	3.44×10^{-5}	6.6×10^{4}	2.83×10^{15}

[a] Particle density n; average molecular spacing d'; mean free path λ; and particle flux on a surface Γ, for $T = 22°C$.

tabulated in Table 2.1 for pressures ranging from atmospheric to ultrahigh vacuum. The pressure dependence of the mean free path is given for several gases in Appendix B.1.

2.2 GAS LAWS

Kinetic theory, as expressed in (2.13), summarizes all the earlier experimentally determined gas laws. However, we review several of the experimentally verified laws here, because they are especially helpful to those with no experience in gas kinetics. When using kinetic theory, we need to remember that the primary assumption of a gas at rest in thermal equilibrium with its container is not always valid in practical situations. For example, a pressure gauge close to and facing a high vacuum cryogenic pumping surface will register a lower pressure than when it is close to and facing a warm surface in the same vessel [1]. This and other non-equilibrium situations will be discussed as required.

2.2.1 Boyle's Law

In 1662 Robert Boyle demonstrated that the volume occupied by a given quantity of gas varied inversely as its pressure, when the gas temperature remained the same.

$$P_1V_1 = P_2V_2 \quad (N,T \text{ constant}) \tag{2.14}$$

This is easily derived from the general law by multiplying both sides by the volume V and noting that $N = nV$.

2.2.2 Amontons' Law

Amontons discovered the pressure in a confined chamber increased as the temperature increased. Amontons' law can be expressed as

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (N, V \text{ constant}) \tag{2.15}$$

In 1703 he constructed an air thermometer based on this relationship. This later came to be known as the law of Gay-Lussac.

2.2.3 Charles' Law

The French chemist Charles found in 1787 that gases expanded and contracted to the same extent under the same changes of temperature provided that no change in pressure occurred. Again by the same substitution in (2.13) we obtain

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (N, P \text{ constant}) \tag{2.16}$$

2.2.4 Dalton's Law

Dalton discovered in 1801 that the total pressure of a mixture of gases was equal to the sum of the forces per unit area of each gas taken individually. By the same methods for a mixture of gases, we can develop the relation

$$P_t = nkT = n_1kT + n_2kT + n_3kT + \cdots \tag{2.17}$$

which reduces to

$$P_t = P_1 + P_2 + P_3 + \cdots \tag{2.18}$$

pressures and densities, respectively. Equation (2.18) is called Dalton's law of partial pressures and is valid for pressures below atmospheric [2].

2.2.5 Avogadro's Law

In 1811 Avogadro observed that pressure and number of molecules were proportional for a given temperature and volume:

$$\frac{P_1}{N_1} = \frac{P_2}{N_2} \quad (T, V \text{ constant}) \tag{2.19}$$

Two terms, "standard temperature and pressure" and "mole" often cause confusion. Standard temperature and pressure (STP) conditions refer to a

gas with a temperature of 0°C, at pressure of 1 atmosphere. However, in Chapter 6, we note that some manufacturers of thermal mass flow controllers assume a temperature of 20°C when calibrating flow meters; this is called NTP, or normal temperature and pressure. In SI, Avogadro's number of any gas species is $N_0 = 6.02252 \times 10^{26}$ and occupies a molar volume of $V_0 = 22.4136\ m^3$. Avogadro's number of molecules is known as a mole. In SI, the unit of volume is the m^3 and the unit of mass is the kg. To avoid confusion with cgs units, we will use "kg-mole." For example one kg-mole of oxygen contains 6.02252×10^{26} molecules and weighs 32 kg. Its density at STP is therefore 32 kg/22.4136 m^3 or 1.45 kg/m^3.

2.2.6 Graham's Law

In the nineteenth century Graham studied the rate of effusion of gases through very small holes in porous membranes. He observed the rate of effusion to be inversely proportional to the square root of the density of the gas provided that the pressure and temperature were held constant. Since the density of a gas is proportional to its molecular weight, Grahams' law can be stated as

$$\frac{\text{effusion rate}_a}{\text{effusion rate}_b} = \left(\frac{M_b}{M_a}\right)^{1/2} \quad (P,T \text{ constant}) \tag{2.20}$$

Grahams' law describes how a helium-filled balloon loses its gas more quickly than an air-filled balloon.

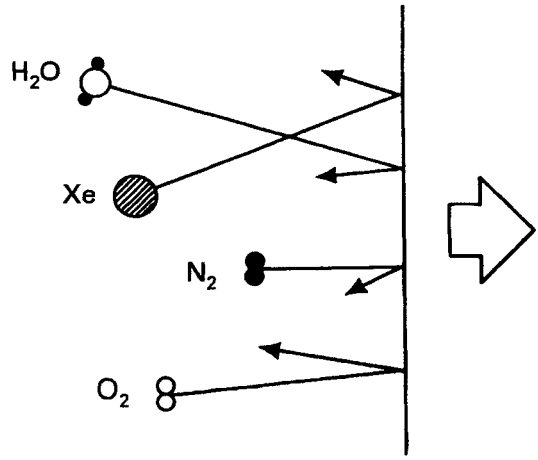

Fig. 2.5 Dalton's law states that the total pressure is the sum of the partial pressures.

2.3 ELEMENTARY GAS TRANSPORT PHENOMENA

In this section approximate views of viscosity, thermal conductivity, diffusion, and thermal transpiration are discussed. We state results from kinetic theory without derivation.

2.3.1 Viscosity

A viscous force is present in a gas when it is undergoing shear. Figure 2.5 illustrates two plane surfaces, one fixed and the other traveling in the x direction with a uniform velocity. The coefficient of absolute viscosity η is defined by the equation

$$\frac{F_x}{A_{xz}} = \eta \frac{du}{dy} \tag{2.21}$$

where F_x is the force in the x direction, A_{xz} is the surface area in the x–z plane, and du/dy is the rate of change of the gas velocity at this position between the two surfaces. Because the gas stream velocity increases as the moving plate is approached, those molecules crossing the plane A_{xz} from below (1 in Fig. 2.5) will transport less momentum across the plane than will those crossing the same plane from above (2 in Fig. 2.5). The result is that molecules crossing from below the plane will, on the average, reduce the momentum of the molecules from above the plane, in the same manner molecules crossing from above the plane will increase the momentum of those molecules below the plane. To an observer this viscous force appears to be frictional; actually it is not. It is merely the result of momentum transfer between the plates by successive molecular collisions. Again, from kinetic theory the coefficient of viscosity is

$$\eta = \frac{1}{3} nmv\lambda \tag{2.22}$$

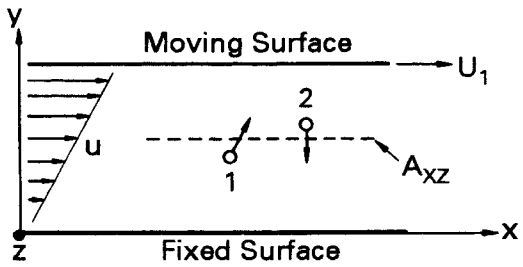

Fig. 2.6 Origin of the viscous force in a gas.

When the gas density is measured in units of m^{-3}, the molecular mass in kg, the velocity in m/s and the mean free path in m, η will have units of (N-s)/m^2, or Pa-s. One Pa-s is equal to 10 poise. A more rigorous treatment of viscosity [3] yields a result with a slightly different numerical coefficient:

$$\eta = 0.499nmv\lambda \qquad \blacktriangleright(2.23)$$

Substituting (2.2) and (2.4) into this result yields

$$\eta = \frac{0.499(4mkT)^{1/2}}{\pi^{3/2}d_o^2} \qquad (2.24)$$

From (2.24) we see that kinetic theory predicts that viscosity should increase as $(mT)^{1/2}$ and decrease as the square of the molecular diameter. An interesting result of this simple theory is that viscosity is independent of gas density or pressure. This theory, however, is valid only in a limited pressure range. If there were a perfect vacuum between the two plates, there would be no viscous force because there would be no mechanism for transferring momentum from one plate to another. This understanding leads to the conclusion that (2.24) is valid as long as the distance between the plates is greater than the mean free path—for example, the gas is viscous.

For a rarefied gas in which the ratio of the mean free path to plate separation $\lambda/y >> 1$ the viscous force can be expressed as

$$\frac{F}{A_{xz}} = \left(\frac{Pmv}{4kT}\right)\frac{U_1}{\beta} \qquad (2.25)$$

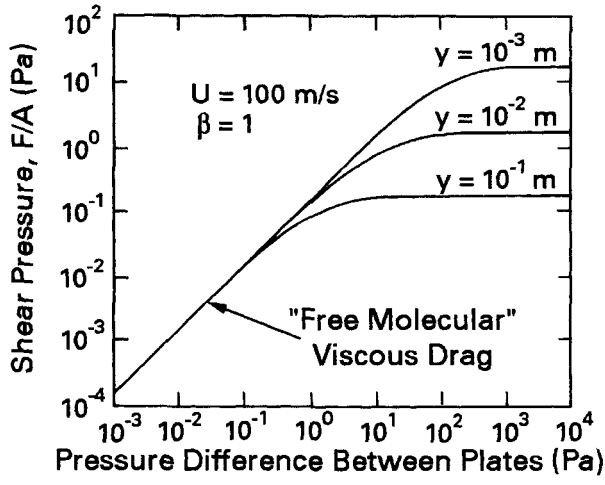

Fig. 2.7 Viscous shear force between two plates at 22°C.

where the term in parentheses is referred to as the free-molecular viscosity. The free-molecular viscosity is directly proportional to the molecular density ($n = P/kT$) available to transfer momentum between the plates. It is valid in the region $\lambda >> y$. The constant β in (2.25) is related to the slip of molecules on the plate surfaces. For most vacuum conditions $\beta \sim 1$.

Figure 2.6 illustrates the magnitude of the viscous force caused by air at 22°C between two plates moving with a relative velocity of 100 m/s for three plate separations. Equation (2.24) was used to calculate the asymptotic value of the viscous drag at high pressures and (2.25) was used to calculate the free molecular limit. A more complete treatment of the intermediate or viscous slip region is given elsewhere [4]. The viscous shear force is independent of the plate spacing as long as the mean free path is larger than the spacing. This idea was used by Langmuir [5] to construct a viscosity gauge in which damping was proportional to pressure.

2.3.2 Thermal Conductivity

Heat conductivity between two infinite parallel plates is explained by kinetic theory in a manner analogous to that used to explain viscosity. The diagram in Fig. 2.6 could be relabeled to make the top plate stationary at temperature T_2, and the lower plate a stationary plate at a temperature T_1, where $T_1 < T_2$. Heat conduction can be modeled by noting that the molecules moving across the plane toward the hotter plate carry less energy than those moving across the plane toward the cooler surface. The heat flow can be expressed as

$$H = AK\frac{dT}{dy} \tag{2.26}$$

where H is the heat flow and K is the heat conductivity. The simple theory predicts that the heat conductivity K is expressed by $K = \eta c_v$, where η is the viscosity and c_v is the specific heat at constant volume. This simple theory is correct only to an order of magnitude. A more detailed analysis, which accounts for molecular rotational and vibrational energy, yields

$$K = \frac{1}{4}(9\gamma - 5)\eta c_v \tag{2.27}$$

where γ is the ratio of specific c_p/c_v. When η has the units of Pa-s and c_v has units (J/kg)/K, then K will have units of (W/m)/K. At room temperature, the heat conductivity and viscosity both increase as $(mT)^{1/2}/d^2$. For infinite parallel plates, K does not depend on pressure, as long as the mean free path is smaller than the plate spacing. In the low-pressure region, the heat transfer between parallel plates [6] has been calculated as

$$E_o = \alpha\Lambda P(T_2 - T_1)$$

where

$$\alpha = \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2 - \alpha_1\alpha_2} \qquad \Lambda = \frac{1}{8}\frac{(\gamma+1)v_1}{(\gamma-1)T_1} \tag{2.28}$$

This equation has the same general form as (2.25) for free-molecular viscosity. Λ is the free-molecular heat conductivity and α_1, α_2, α are the accommodation coefficients of the cold surface, hot surface, and system, respectively. If molecules can thermally equilibrate with the surface, say by making many small collisions on a rough surface, α will have a value that approaches unity. If however, the same surface is smooth and the molecule recoils without gaining or losing energy, α will approach zero. The kinetic picture of heat conductivity is rather like viscosity except that *energy transfer* determines the thermal conductivity, and *momentum transfer* determines the viscous drag. Even so, Fig. 2.5 can be sketched for the thermal conductivity of a gas between two parallel plates, where the vertical axis has dimensions of heat flow. In SI Λ has units of $W\text{-}m^{-2}\text{-}K^{-1}\text{-}Pa^{-1}$, whereas E_o has units of W/m^2. Tables of the accommodation coefficient are given elsewhere [3,7]. The accommodation coefficient of a gas is not only dependent on the material but on its cleanliness, surface roughness, and gas adsorption as well.

When the heated parallel plate is replaced by a heated fine wire, the situation changes. In the case of a heated fine wire, the upper "knee" of the curve is not dependent on the ratio of λ to plate separation, but rather on the ratio of λ to hot wire diameter [8,9]. Both thermocouple and Pirani gauges operate in a region in which the heat conduction from the heated wire is linearly dependent on pressure. However, the "knee" in their linear range begins at a mean free path equal to a few multiples of the wire diameter and not at a mean free path related to the wire-envelope distance.

2.3.3 Diffusion

Diffusion is a complex phenomenon. This discussion has been simplified by restricting it to the situation in a vessel that contains two gases whose compositions vary slowly throughout the vessel but whose total number density is everywhere the same. The coefficient of diffusion D, of two gases is defined in terms of their particle fluxes $\Gamma_{1,2}$

$$\Gamma_1 = -D\frac{dn}{dx} \qquad \Gamma_2 = -D\frac{dn_2}{dx} \tag{2.29}$$

These fluxes result from the partial pressure gradient of the two gases. The result from kinetic theory, when corrected for the Maxwellian distribution of velocities and for velocity persistence, is [9]

$$D_{12} = \frac{8\left(\frac{2kT}{\pi}\right)^{1/2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)^{1/2}}{3\pi(n_1+n_2)(d_{o1}+d_{o2})^2} \qquad \blacktriangleright(2.30)$$

where D_{12} is the constant of inter-diffusion of the two gases. In SI it has units of m^2/s. For the case of self diffusion, the coefficient is

$$D_{11} = \frac{4}{3\pi n d_o^2}\left(\frac{kT}{\pi m}\right)^{1/2} \qquad \blacktriangleright(2.31)$$

If the density n is replaced by P/kT, it becomes apparent that the diffusion constant is approximately proportional to $T^{3/2}$ and P^{-1}.

The diffusion equation, $dC/dt = -Dd^2C/dt^2$, whose solutions we do not describe here, contains the term $(Dt)^{1/2}$. This term has the dimensions of length and is called "diffusion length." For long times the diffusion "front" moves through the gas in proportion to $(Dt)^{1/2}$. Values of the diffusion constant for several gases in air are given in Appendix B.2.

Examination of (2.31) shows the diffusion coefficient will become infinitely large as the density of molecules goes to zero. This does not happen. When the pressure becomes low enough so that the mean free path is much larger than the dimensions of the container, say the diameter of a pipe, gas diffusion is limited by molecules recoiling from walls, rather than from collisions with each other. At low pressures, where $\lambda >> d$, the diffusion coefficient is given by

$$D = \frac{2}{3} rv \qquad \blacktriangleright(2.35)$$

where r is the radius of the pipe and v is the thermal velocity. This is called the Knudsen diffusion coefficient for a long capillary [10].

2.3.4 Thermal Transpiration

When a tube or orifice connects two chambers of different temperatures, their relative pressures depend on the nature of the gas flow in the connecting tubing. The nature of the gas flow in the tubing or orifice is characterized by (λ/d), where d is the diameter of the connecting tube or orifice. For $\lambda << d$ the pressure is everywhere the same in both chambers, $P_1 = P_2$. The densities in the two chambers are related by

$$\frac{n_2}{n_1} = \frac{T_1}{T_2} \qquad \blacktriangleright(2.36)$$

When the orifice diameter is such that $\lambda >> d$, the flux of gas through the orifice is given by (2.8).

$$\Gamma_{1,2} = \frac{n_{1,2}}{4}\left(\frac{8kT_{1,2}}{\pi m}\right)^{1/2} = \frac{P_{1,2}}{(2kT\pi m)^{1/2}} \qquad \blacktriangleright(2.37)$$

In steady state the net flux between the two chambers must be zero—for example, ($\Gamma_{1\to2} = \Gamma_{2\to1}$)—with the result

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{1/2} \qquad \blacktriangleright(2.38)$$

Equations (2.36) and (2.38) can be used to calculate the pressures within furnaces or cryogenic enclosures when the pressure gauge is located outside the enclosure at a different temperature. Equation (2.36) is used at high pressure ($\lambda < d/10$), and (2.38) is used at low pressure ($\lambda > 10d$).

Thermal transpiration was discovered by Neumann [11] and studied by Maxwell [12], who predicted the square-root dependence given in (2.38). The geometry and reflectivity from walls of the connecting tubing introduce deviations from the theory. Siu [13] has studied these effects and has predicted that (2.38) is obtained in short tubes only for specular reflection and in long tubes only for diffuse reflection.

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PROBLEMS

2.1 † State the assumptions that form the basis of kinetic theory.

2.2 Consider a 1-cm-diameter pipe 10 meters long. (a) Sketch the self-diffusion constant of air in this pipe over the pressure range 0.01 Pa to 10^5 Pa. (b) Knowing that the diffusion front moves as $d^2 \propto Dt$, estimate the time required for air to diffuse to the end of the 10-meter-long pipe over the pressure range given in (a).

2.3 The diffusion constant for gas (1) in gas (2), D_{12} describes how gas a gas of one molecular weight diffuses in the background of a second gas. For the case where the diffusing gas (1) is light ($m_1 << m_2$), and is present in a small concentration ($n_1 << n_2$), derive a simplified relationship for (2.30). This happens during helium leak checking.

2.4 † Room temperature N_2 molecules are directed toward a surface 100-cm distant. To what pressure must a chamber be evacuated in order that the molecules reach the surface, on average, without first colliding with another N_2 molecule?

2.5 † What is the advantage of low-pressure chemical vapor deposition over atmospheric pressure chemical vapor deposition?

2.6 Explain why the viscosity of a gas should increase with increasing particle mass and temperature. (The viscosity of a liquid decreases with increasing temperature.)

2.7 † The air surrounding it cools an object that is heated by a constant-power source. What would happen to its temperature, if the surrounding air were replaced (a) by helium and (b) by argon?

2.8 What is the heat flow by thermal conduction between two 0.1 m^2 sheets of copper with a temperature difference of 100°C and which are separated by 0.1 cm of CO_2 at pressures of (a) 10^5 Pa, (b) 10^3 Pa, and (c) 100 Pa.

2.9 A 0.5-cm-diameter tube interconnects two chambers. The left-hand chamber is heated to a temperature of 250°C, while the right-hand chamber remains at room temperature. Over what pressure range is the pressure in the two chambers the same? Over what pressure range does the transpiration equation apply? Sketch a plot of P_{hot}/P_{cold} versus the pressure of the cold chamber. Assume diffuse scattering.

2.10 In Problem 2.9, nitrogen is replaced by helium. Sketch a plot of P_{hot}/P_{cold} versus the pressure of the cold chamber in the transition region. How does this compare to the curve in Problem 2.9 for helium?

CHAPTER 3

Gas Flow

In this chapter we discuss the flow of gas at reduced pressures, as it is encountered in a vacuum system. Gas flow is complex and the nature of the solution depends on the flow rate and gas properties as well as the geometry and surface properties of the duct. We begin by defining the flow regimes and introducing the concepts of throughput, mass flow and conductance. We describe the gas throughput and conductance for several kinds of flow. We show how approximation techniques and probability methods are used to solve complex problems, such as flow in ducts containing entrance and exit orifices, aperture plates, or other irregular shapes.

3.1 FLOW REGIMES

Gas flow regimes are characterized by the *nature* of the gas and by the *relative quantity* of gas flowing in a pipe. The nature of the gas is determined by examining Knudsen's number, whereas Reynolds' number describes the relative flow. In the viscous gas region (high pressures) the flow is called continuum flow. The flow can be further described as turbulent or viscous. Turbulent flow is chaotic, like the flow behind a moving vehicle or the rising smoke some distance from a cigarette. Laminar or stream flow occurs when the velocity and surface irregularities are small enough for the gas to flow gently past obstructions in laminar streamlines. In the molecular gas region, the mean free path is so long in comparison to the pipe size that the flow is entirely determined by gas–wall collisions. The flow in this region is called molecular flow. Between the continuum flow region and the molecular flow region is the transition region. In this region gas molecules collide with each other and with walls.

A viscous gas is characterized by a Knudsen number of < 0.01. Knudsen's number Kn, is a dimensionless ratio of the mean free path to a characteristic dimension of the system, say, the diameter of a pipe:

$$\mathrm{Kn} = \frac{\lambda}{d} \qquad \blacktriangleright (3.1)$$

In continuum flow the diameter of the pipe is much greater than the mean free path and the character of the gas flow is determined by gas–gas collisions. The flow has a maximum velocity in the center of the channel and zero velocity at the wall. Continuum flow can be either turbulent or laminar viscous. The boundary between turbulent and viscous flow can be expressed in terms of Reynolds' dimensionless number **R** for round pipes:

$$\mathbf{R} = \frac{U\rho d}{\eta} \qquad (3.2)$$

where ρ is the mass density (kg/m^3), of the gas of viscosity η flowing with stream velocity U in a pipe of diameter d. Reynolds' number is used to characterize the relative quantity of gas flow. It is a ratio of the shear stress due to turbulence to the shear stress due to viscosity. Alternatively, it tells something about the forces necessary to drive a gas system in relation to the forces of dissipation due to viscosity. Reynolds [1] found two flow situations dynamically similar when this dimensionless number was the same. When $\mathbf{R} > 2200$, the flow was always turbulent and when $\mathbf{R} < 1200$ the flow was always viscous [2]. In the region $1200 < \mathbf{R} < 2200$ the flow was viscous or turbulent, depending on the geometry of the inlet and outlet and on the nature of the piping irregularities.

Laminar viscous flow, the ordered flow of a gas in streamlines, occurs in the region bounded by a Reynolds' number lower than 1200 and a Knudsen number less than 0.01.

When the mean free path is equal to or greater than the pipe diameter, say $\mathrm{Kn} > 1$, and when $\mathbf{R} < 1200$, the gas is said to be a molecular gas, and the flow is called molecular flow. To be precise, Reynolds' number does not have any meaning for a gas in the free-molecular regime, because classical viscosity cannot be defined. The nature of molecular flow is very different from laminar viscous flow. Gas–wall collisions predominate and the concept of viscosity is meaningless. For most surfaces, diffuse reflection at the wall is a good approximation; that is, each particle arrives, sticks, rattles around in a surface imperfection, and is re-emitted in a direction independent of its incident velocity. Thus there is a chance that a particle entering a pipe in which $\lambda \gg d$ will not be transmitted, but will be returned to the entrance. In molecular flow, gas molecules do not collide with one another, and gases can flow in opposite directions without interaction.

In the region $1 > \mathrm{Kn} > 0.01$ the gas is neither viscous nor molecular. Flow in the transition region is difficult to treat theoretically. In this range,

called the transition, or slip flow range, where the pipe is several mean free paths wide, the velocity at the wall is not zero, as in viscous flow and the reflection is not diffuse, as in free molecular flow. Now let us define throughput, mass flow and conductance and develop some practical gas flow formulas.

3.2 THROUGHPUT, MASS FLOW, AND CONDUCTANCE

Throughput is the quantity of gas (the volume of gas at a known pressure) that passes a plane in a known time; $d/dt(PV) = Q$. In SI throughput has units of Pa-m^3/s. Because 1 Pa = 1 N/m^2, and 1 J = 1 N-m, the units could be expressed as J/s or watts (1 Pa-m^3/s = 1 W). Throughput is the energy per unit time crossing a plane. The energy in question is not the kinetic and potential energy contained in the gas molecules, but rather the energy required to transport the molecules across a plane. Expressing gas flow in units of watts is awkward and not used, but it helps to explain the concept that throughput is energy flow. Throughput is a volumetric dimension (volume of gas/unit time). Throughput cannot be converted to mass flow unless the temperature is specified. It is in many ways unfortunate that vacuum technologists have chosen to use a volumetric unit, which conveys incomplete information. Volumetric flow does not conserve mass.

Mass flow, molar flow, or molecular flow are, respectively, the quantity of substance in units of kg, kg-moles, or molecules that passes a plane in a known time. Equation (3.3) describes the relationship between molar flow and throughput.

$$\mathbf{N'(kg\text{-}mole/s)} = \frac{Q}{N_o kT} = \frac{Q}{RT} \tag{3.3}$$

In a similar fashion, mass flow is related to throughput by N'(kg/s) = $MQ/N_o kT$. Throughput can be related to molar or mass flow, only if the temperature is constant and known. A spatial change in the temperature can alter the throughput without altering the mass flow. We discuss applications of mass flow in Chapter 6 (flow meters) and in Chapter 15 where we describe cryogenically pumped systems.

The flow of gas in a duct or pipe is dependent on the pressure drop across the object as well as its cross-sectional geometry. Division of the throughput by the pressure drop across a duct held at constant temperature yields a property known as the conductance of the duct.

$$C = \frac{Q}{P_2 - P_1} \tag{3.4}$$

In SI the unit of throughput is the Pa-m^3/s and the unit of conductance or pumping speed is the m^3/s; however, related throughput units of Pa-L/s and conductance units of L/s are widely used. Unless explicitly stated, all formulas in this chapter use the cubic meter as the volumetric unit.

The pressures $P_1$and P_2 in (3.4) refer to the pressures measured in large volumes connected to each end of the channel or component. According to (3.4) conductance is the property of the object between the points at which the two pressures are measured. For those whose first introduction to flow was with electricity (3.4) is analogous to an electrical current divided by a potential drop. As with electrical charge flow, there are situations (transition, viscous and choked flow) in which the gas conductance is nonlinear, that is, a function of the pressure in the tube. Unlike electrical charge flow, there are cases in which the molecular conductance depends not only on the object, but also on the nature of adjacent objects and how they allow particles to be diffusely scattered from their surfaces. We will explore this last issue in detail when we describe methods for combining conductances in the molecular flow regime.

3.3 CONTINUUM FLOW

A gas is called a viscous gas when Kn < 0.01. The flow in a viscous gas can be either turbulent **R** > 2200, or viscous **R** < 1200. Equation (3.2) can be put in a more useful form by replacing the stream velocity with

$$U = \frac{Q}{AP} \tag{3.5}$$

If we replace the mass density, using the ideal gas law, (3.2) becomes

$$\mathbf{R} = \frac{4m}{\pi kT\eta}\frac{Q}{d} \tag{3.6}$$

For air at 22°C, this reduces to

$$\mathbf{R} = 8.41\times10^{-4}\frac{Q(\text{Pa - L/s})}{d} \tag{3.7}$$

In ordinary vacuum practice turbulent flow occurs infrequently. Reynolds' number can reach high values in the piping of a large roughing pump during the initial pumping phase. For a pipe 250 mm in diameter connected to a 47-L/s pump, **R** at atmospheric pressure is 16,000. Turbulent flow will exist whenever the pressure is greater than 1.5×10^4 Pa (100 Torr). In practice, roughing lines are often throttled during the initial portion of the roughing cycle to prevent the sudden out-rush of gas from scattering

process debris that may reside on the chamber floor. The flow in the throttling orifice is turbulent at high pressures.

In the high flow limit of the turbulent flow region the velocity of the gas may reach the velocity of sound in the gas. Further reduction of the downstream pressure cannot be sensed at the high-pressure side so that the flow is choked or limited to a maximum or critical value of flow. The value of critical flow depends on the geometry of the element, for example, orifice, short tube, or long tube, and the shape of the entrance. A detailed discussion of critical flow has been given by Shapiro [3].

Rather than divide the discussion of continuum flow into viscous, turbulent and critical, it is easier to discuss the flow in terms of the geometry of the pipe. We divide this discussion into orifice flow, long tube flow, and short tube flow, and we give equations for each region.

3.3.1 Orifices

For tubes of zero length (an extremely thin orifice) the flow versus pressure is a rather complicated function of the pressure. Consider a fixed high pressure, say atmospheric pressure, on one side of the orifice with a variable pressure on the downstream side. As the downstream pressure is reduced, the gas flowing through the orifice will increase until it reaches a maximum. At this ratio of inlet to outlet pressure (the critical pressure ratio), the gas is flowing at the speed of sound in the gas. The gas flow through the orifice is given by

$$Q = AP_1C'\left(\frac{2\gamma}{\gamma-1}\frac{kT}{m}\right)^{1/2}\left(\frac{P_2}{P_1}\right)^{1/\gamma}\left[1-\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}\right]^{1/2} \qquad \blacktriangleright(3.8)$$

$$\text{for } 1 > P_2/P_1 \geq (2/\gamma+1))^{\gamma/(\gamma-1)}$$

The factor C' accounts for the reduced cross-sectional area as the high-speed gas stream continues to decrease in diameter, after it passes through the orifice. This phenomenon is called the vena contracta. For thin, circular orifices, C' is ~0.85. If the downstream pressure P_2 is further reduced, the gas flow will not increase, because the gas in the orifice is traveling at the speed of sound and cannot communicate with the high-pressure side of the orifice to tell it that the pressure has changed. In this region P_2 cannot influence the flow so long as $P_2/P_1 < (\ 2/(\lambda+1)\ )^{\lambda/(\lambda-1)}$. The ratio of specific heats is λ whose values are given in Appendix B.4. The flow is given by

$$Q = AP_1C'\left(\frac{kT}{m}\frac{2\gamma}{\gamma+1}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \qquad \blacktriangleright(3.9)$$

$$\text{for } P_2/P_1 \leq (2/(\gamma+1))^{\gamma/(\gamma-1)}$$

This value is called critical, or choked, flow. See Fig. 3.1. This limit is important in describing flow restrictors (devices that control gas flow and the rate of pumping or venting in a vacuum system), choked flow in air-to-air load locks, and flow through small leaks from atmosphere. In any of these relationships, the conductance can be found from $C = Q/(P_1\text{-}P_2)$. For air at 22°C, $\lambda = 1.4$ and $P_2/P_1 = 0.525$; the choked-flow limit is

$$Q(\text{Pa - m}^3/\text{s}) = 200 P_1 A C' \quad \text{for air at } 22^\circ\text{C, when } P_2/P_1 \le 0.52 \qquad \blacktriangleright(3.10)$$

$$Q(\text{Pa - L/s}) = 2\times10^5 P_1(\text{Pa}) A(\text{m}^2) C' \quad \text{for air at } 22^\circ\text{C, when } P_2/P_1 \le 0.52 \qquad \blacktriangleright(3.11)$$

3.3.2 Long Round Tubes

A general mathematical treatment of viscous flow results in the Navier-Stokes equations, which are most complex to solve. The simplest and most familiar solution for long straight tubes is the equation due to independently to Poiseuille and Hagen, and called the Hagen-Poiseuille equation:

$$Q = \frac{\pi d^4}{128\eta l}\frac{(P_1+P_2)}{2}(P_1-P_2) \qquad \blacktriangleright(3.12)$$

The gas flow for air at room temperature becomes

$$Q(\text{Pa - m}^3/\text{s}) = 718.5\frac{d^4(P_1+P_2)}{l}(P_1-P_2) \qquad (3.13)$$

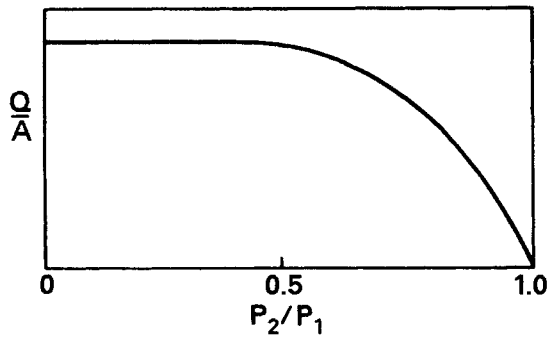

Fig. 3.1 Throughput versus pressure ratio in a circular orifice.

This specific solution is valid when four assumptions are met: (1) fully developed flow (the velocity profile is not position-dependent), (2) laminar flow, (3) zero wall velocity, and (4) incompressible gas. Assumption 1 holds for long tubes in which the flow lines are fully developed. The criterion for fully developed flow was determined by Langhaar [4] who showed that a distance of $l_e = 0.0568d\mathbf{R}$ was required before the flow streamlines developed into their parallel, steady-state profile. For air at 22°C this reduces to l_e (meters) = $0.0503Q$ when Q is given in units of Pa-m^3/s. Assumptions 2 and 3 are satisfied if $\mathbf{R} < 1200$ and if Kn < 0.01. The assumption of incompressibility holds true, provided that the Mach number U, the ratio of gas-to-sound velocity, is < 0.3.

$$\mathbf{U} = \frac{U}{U_{sound}} = \frac{4Q}{\pi d^2 P U_{sound}} < \frac{1}{3} \tag{3.14}$$

For the special case of air at 22°C we have

$$Q(\text{Pa - L/s}) < 9.0 \times 10^5 d^2 P \qquad \blacktriangleright (3.15)$$

This is a value of flow that may be exceeded in many cases and would render the results of the Poiseuille equation incorrect.

Relationships for viscous flow between long, coaxial cylinders and long tubes of elliptical, triangular and rectangular cross section have been tabulated by Holland et al. [5]. Williams et al. [6] give the relation for flow in a long rectangular duct for air at 20°C

$$Q(\text{Pa - L/s}) = 4.6Y \frac{b^2(\text{cm}^2)h^2(\text{cm}^2)}{l(\text{cm})} \frac{(P_1 + P_2)}{2}(P_1 - P_2) \tag{3.16}$$

where the duct cross-section dimensions b and h and the length l are given in cm. The function $Y(h/b)$ is obtained from the following table:

h/b	Y	h/b	Y	h/b	Y
1.0	0.4217	0.4	0.30	0. 05	0.0484
0.8	0.41	0.2	0.175	0.02	0.0197
0.6	0.31	0.1	0.0937	0.01	0.0099

In the limit $h << b$, the air flow reduces to the one-dimensional solution of Sasaki and Yasunaga [7]

$$Q(\text{Pa - L/s}) = 4.6Y \frac{h^3(\text{cm}^3)b(\text{cm})}{l(\text{cm})} \frac{(P_1 + P_2)}{2}(P_1 - P_2) \tag{3.17}$$

Again h, b, and l are given in cm. The flow in (3.16) and (3.17), like (3.13), is inversely proportional to viscosity and may be accordingly scaled for other gases. These relations for long tubes are of limited use. They are of use in components such as mass flow meter tubes, controlled leaks, and piping that connects chambers with remotely located pumps and gas tanks. In most practical cases we connect chambers with as short a duct as possible to reduce unwanted pressure drops, and we need to know relationships which are valid for these cases.

3.3.3 Short Round Tubes

As we noted above, the flow in short tubes does not obey the Poiseuille equation. The flow may switch from viscous to to critical flow without there being any pressure region in which the Poiseuille equation is valid. This problem has been treated in several ways. Dushman [8] gives a non-linear relation for flow in short round tubes. It is valid only for unchoked flow. Santeler [9] devised a technique in which he models the short tube as an aperture in series with a short tube of length l'. The problem is formulated by assuming an unknown pressure P_k between the "tube" and the "aperture." This is the pressure that would be measured by a gauge just inside the end of the tube that was pointing upstream. Figure 3.2 illustrates an application of this technique—calculating the pressure drop and airflow through a 100-μm-diameter leak in a 1-cm-thick vacuum wall. Santeler's model uses (3.13) with P_1 replaced by P_x. The flow through the aperture was modeled using (3.10) with P_x as its inlet pressure, and "high vacuum" ($P = 0$) as its outlet pressure. Since the two flows are in series, they are equal; the solution is $P_x = 44{,}560$ Pa. The answer can be checked to ensure that the assumption of choked flow in the aperture is valid; if not, then (3.8) must be used in place of (3.10). This model predicts Poiseuille leak flow with significant gas expansion at the vacuum side.

3.4 MOLECULAR FLOW

A gas is called a molecular gas when Kn > 1.0. This is equivalent to stating that $Pd < 6.6$ Pa-mm (4.95 Torr-mm) for air at 22°C. In this region the flow is called molecular flow. For completeness we could say that $\mathbf{R} < 1200$; however, we cannot define a Reynolds number in the region where viscosity cannot be defined. The molecular flow region is theoretically the best understood of any flow type. This discussion focuses on orifices, infinite tubes, finite tubes, and other shapes, including combinations of components in molecular flow.

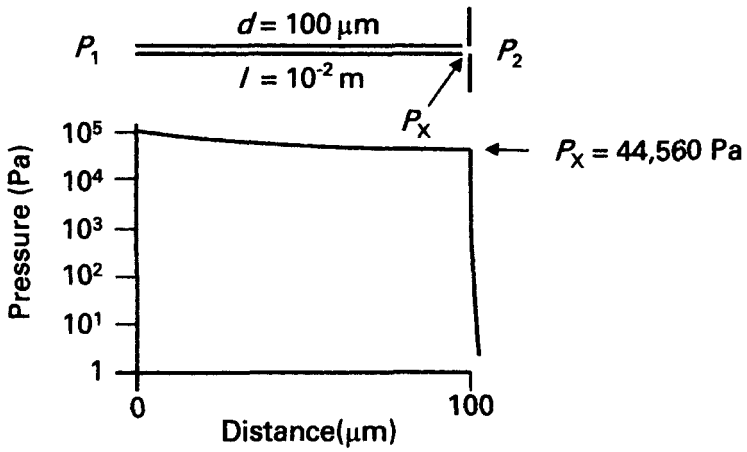

Fig. 3.2 Pressure profile through a fine leak in a vacuum wall, as calculated with Santeler's model. This model assumes Poiseuille flow through the tube with a precipitous drop in pressure immediately within the vacuum vessel caused by choked flow at the exit.

3.4.1 Orifices

If two large vessels are connected by an orifice of area A and the diameter of the orifice is such that Kn > 1, then the gas flow from one vessel (P_1, n_1) to the second vessel (P_2, n_2) is given by

$$Q = \frac{kT}{4} vA(n_1 - n_2) = \frac{v}{4} A(P_1 - P_2) \tag{3.18}$$

and the conductance of the orifice is

$$C = \frac{Q}{P_1 - P_2} = \frac{v}{4} A \tag{3.19}$$

which for air at 22°C has the value

$$C(\mathrm{m}^3/s) = 116A(\mathrm{m}^2) \tag{3.20}$$

or

$$C(\mathrm{L/s}) = 11.6A(\mathrm{cm}^2) \tag{3.21}$$

From (3.18) we note an interesting property of the molecular flow regime. Gas can flow from vessel 2 to vessel 1; at the same time gas is flowing from vessel 1 to vessel 2 without either of the gases colliding with gas that originated in the other vessel.

3.4.2 Long Round Tubes

The diffusion method of Smoluchowski [10] and the momentum transfer method of Knudsen [11] and Loeb [12] were the first used to describe gas flow through very long tubes in the free molecular flow region. For circular tubes both derivations yield conductances of

$$C_{\text{tube}} = \frac{\pi}{12} v \frac{d^3}{l} \tag{3.22}$$

For air at 22°C this becomes

$$C_{\text{tube}}(\text{m}^3/\text{s}) = 121 \frac{d^3}{l} \tag{3.23}$$

Conductance relations for long noncircular tubes have been derived [13].

3.4.3 Short Round Tubes

The flow equation for long tubes (3.22) indicates the conductance becomes infinite as the length tends toward zero, whereas in Section 3.4.1 we showed the conductance actually becomes $vA/4$. Dushman [14] developed a solution to the problem of short tubes by considering the total conductance to be the sum of the reciprocal conductances of an aperture and a section of tube of length l.

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_{\text{tube}}} + \frac{1}{C_{\text{aperture}}} \tag{3.24}$$

As $l/d \to 0$, (3.24) reduces to (3.19), and as $l/d \to \infty$ it reduces to (3.22). Although this equation gives the correct solution for the extreme cases, it is not correct for the intermediate. It can be in error by as much as 12–15%.

The difficulty in performing calculations for short tubes lies in the nature of the gas–wall interaction. Lorentz [15] assumed the walls of a pipe are molecularly rough; that is, molecules are scattered according to the cosine law (diffuse reflection). Molecules hit a wall, oscillate in potential wells, and recoil in a direction that is independent of their arrival angle. In diffuse reflection, scattered molecules have the greatest probability of recoiling at an angle of 90° from the surface. Particles not scattering at 90° have as much likelihood of going forward through the tube as going backward toward the source. See Fig. 3.3. Clausing [16] solved this problem by calculating the probability that a molecule entering the pipe at one end will escape at the other end after making diffuse collisions with the walls. Clausing's solution is in the form of an integral that is difficult to evaluate. For simple cases such as round pipes, Clausing and others have generated approximate solutions. The solution has been tabulated in many standard